

TESTS OF DIVISIBILITY OF NUMBERS (ShortCuts)

1. Divisibility By 2 : A number is divisible by 2, if its unit's digit is any of 0, 2, 4, 6, 8. Ex. 84032 is divisible by 2, while 05935 is not.

Consider a two digit number $(10*a+b)$. Factor this to $2*5*a + b$. This shows that all numbers ending a zero are divisible by two, so if the ones digit is divisible by two, the entire number is as well.

2. Divisibility By 3 : A number is divisible by 3, if the sum of its digits is divisible by 3.

Consider a 2 digit number $10*a + b = 9*a + (a+b)$. We know that $9*a$ is divisible by 3, so $10*a + b$ will be divisible by 3 if and only if $a+b$ is.

Similarly, $100*a + 10*b + c = 99*a + 9*b + (a + b + c)$, and $99*a + 9*b$ is divisible by 3, so the total will be divisible by 3 if and only if $a + b + c$ is.

Ex. 592482 is divisible by 3, since sum of its digits = $(5 + 9 + 2 + 4 + 8 + 2) = 30$, which is divisible by 3. But, 804329 is not divisible by 3, since sum of its digits = $(8 + 0 + 4 + 3 + 2 + 9) = 32$, which is not divisible by 3.

3. Divisibility By 4 : A number is divisible by 4, if the number formed by the last two digits is divisible by 4.

Consider a three digit number $(100*a + 10*b + c)$
Factor the first digit: $4*25*a + 10*b + c$

Ex. 892648 is divisible by 4, since the number formed by the last two digits is 48, which is divisible by 4. But, 749282 is not divisible by 4, since the number formed by the last two digits is 82, which is not divisible by 4.

4. Divisibility By 5 : A number is divisible by 5, if its unit's digit is either 0 or 5.

Factor the first digit: $5*2*a + b$

Thus, 20820 and 60345 are divisible by 5, while 30934 and 40946 are not.

5. Divisibility By 6 : A number is divisible by 6, if it is divisible by both 2 and 3. Es.

The number 35258 is clearly divisible by 2. Sum of its digits = $(3 + 5 + 2 + 5 + 8) = 21$, which is divisible by 3.

Thus, 35256 is divisible, by 2 as well as 3. Hence, 35256 is divisible by 6.

6. Divisibility By 8 : A number is divisible by 8, if the number formed by the last three digits of the given number is divisible by 8.

Factor the first digit: $8*125*a + 100*b + 10*c + d$

Since we know $8*125*a$ is divisible by 8, we only need to consider $100*b + 10*c + d$, the last three digits, for divisibility by 8.

Ex. 953360 is divisible by 8. since the number formed by last three digits is 360, which is divisible by 8. But, 529418 is not divisible by 8. since the number formed by last three digits is 418, which is not divisible by 8.

7. Divisibility By 9 : A number is divisible by 9, if the sum of its digits is divisible by 9.

Ex. 60732 is divisible by 9. since sum of digits $(6 + 0 + 4 + 7 + 3 + 2) = 18$. which is divisible by 9.

But, 68956 is not divisible by 9, since sum of digits $(6+8+9+5+6)= 34$. which is not divisible by 9.

8. Divisibility By 10 : A number is divisible by 10. if it ends with 0. Ex. 96410. 10486 are divisible by 10, while 96375 is not.

9. Divisibility By 11 : A number is divisible by 11, if the difference of the sum of its digits at odd places and the sum of its digit. at even places, is either 0 or a number divisible by 11.

ex: The number 4832718 is divisible by 11, since :

(sum of digits at odd places) - (sum of digits at even places)

$(8+7+3+4) - (4 + 2 + 8) = 11$, which is divisible by 11.

10. Divisibility By 12 : A number is divisible by 12. if it is divisible by both 4 and 3.

Ex. Consider the number 34632.

(i) The number formed by last two digits is 32, which is divisible by 4.

(ii) Sum of digits $(3+4+6 + 3 + 2) = 18$. which is divisible by 3.

Thus. 34632 is divisible by 4 as well as 3. Hence, 34632 is divisible by 12.

11. Divisibility By 14 : A number is divisible by 14, if it is divisible by 2 as well as 7.

12. Divisibility By 15 : A number is divisible by 15, if it is divisible by both 3 and 5

13. Divisibility By 16 : A number is divisible by 16, if the number formed by the last 4 digits is divisible by 16. Ex. 7957536 is divisible by 16, since the number formed by the last four digits is 7536, which is divisible by 16.

14. Divisibility By 24 : A given number is divisible by 24, if it is divisible by both 3 and 8.

15. Divisibility By 40 : A given number is divisible by 40, if it is divisible by both 5 and 8.

16. **Divisibility By 80** : A given number is divisible by 80, if it is divisible by both 5 and 16.

17. **Divisibility by a prime number p** : In fact, for any prime p , there exists some integer k such that divisibility by p can be determined by multiplying the units digit by k and adding it to the truncated portion of the numeral.

Divisibility tests can be used to find factors of large whole numbers quickly, and thus determine if they are prime or composite. When working with large whole numbers, tests for divisibility are more efficient than the traditional factoring method.

